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SOME METHODS FOR GENERATING PROXIMTIES BY RELATIONS

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Abstract— In the current paper, a procedure for obtaining clopen topology from a general binary relations R via proximity structures was obtained. A comparison between the properties of approximation with respect to the topologies generated by relation and their corresponding approximation with respect to proximity structures were carried out.

Index Terms—General binary relation, Proximity space, Neighborhoods, Approximation spaces.
Subject Classification: 54C05.

1 INTRODUCTION

Relations are used in the construction of topological structures in many fields such as dynamics [7], rough set theory and approximation space [2, 14, 15], digital topology [19, 20], biochemistry [22] and biology [23]. In principal, topology is a branch of mathematics, whose concepts exist not only in almost all branches of mathematics, but also in many real life applications. It should be noted that the generation of topology by relations and the representation of topological concepts via relations will narrow the gap between topology and its applications. Recently the concept of rough set theory RST ([2, 9, 10, 13, 14, 15, 17, 24]) has a wide range of both applications and theoretical aspects directed attention to the importance of topology in applications. It is well known that this theory is based on the properties of the special type of topological structures, clopen topology. This connection between topology and rough set theory helped topologist to look a topological structures as generalized mathematical models to represent information. In past several years of 21st, rough set theory has developed significantly due to its wide applications. Various generalized rough set models have been established and their properties and structures have been investigated intensively. One of the interesting research topics in RST is to modify this theory via topology. To the best of our knowledge, proximity structures did not take the suitable interest in generalizing rough set models. The purpose of this work is to introduce proximity structures to approximation spaces which is the basic concept in RST. The paper is organized as follow, in section 2 we gave account basic definition and preliminaries, the purpose of section 3 is to initiate proximity structures and give Examples. Also, properties and characterization of the two constructed proximities are given.

2 PRELIMINARIES

Throughout this paper, by P(X) we mean the power set of X. The following are cited from [3, 11, 12]

Definition 2.1 For every subset A and B of X and $x\in X$. A relation $\delta\subseteq X\times P(X)$ is said to be a K-proximity on a given set X if it satisfies the following conditions (where $\bar{\delta}$ means negation of δ):
(P1) $x\delta (A\cup B) \leftrightarrow x\delta A$ or $x\delta B$.
(P2) $x\bar{\delta}\varnothing \ \ \forall x\in X$.
(P3) $x\in A \rightarrow x\delta A$.
(P4) $x\bar{\delta}A\rightarrow\exists E\subseteq X$ s.t. $x\ \bar{\delta}\ E$ and $y\ \bar{\delta}A\ \forall y\in(X-E)$.

A relation δ is said to be separated [10], if it satisfies:
(P5) $x\delta y\rightarrow x=y$.

Definition 2.2 Let (X, δ) be a K-proximity space and $A\subseteq X$then:
(i) A is τ_δ open, if ($x\bar{\delta}A^c\ \forall x\in A$).
(ii) A is τ_δ closed, if ($x\delta A\rightarrow x\in A$).

Definition 2.3 Let (X, δ) be a K-proximity space, then a subset B of X is a δ-neighborhood of x (in symbols $x<B$), if $x\bar{\delta}$ (X-B).

The family $N(\delta, \{x\}) = \{B\subseteq X:\ x<B\}$ is a δ-nbd. system of x.

Also, $N(\delta, x)\subseteq N(\tau_\delta, x)$ where $N(\tau_\delta, x)$ is the nbd. system of x w.r.t. τ_δ.

Lemma 2.4 Let (X, δ) be a K-proximity space and let A and B be subsets of X, then:
(i) $x\delta A,\ A\subseteq B \rightarrow x\delta B$.
(ii) The operator $Cl_{\delta}(A)=\{x\in X\ :\ x\delta A\}$ is a Kuratowski closure operator which produce the topology τ_δ generated by δ.
(iii) $\{x\}\delta A\rightarrow Cl_\delta\{x\}\delta Cl_\delta A$, where the closure is taken w.r.t. τ_δ.

3 PROXIMITY STRUCTURES ON GENERALIZED APPROXIMATION SPACES

The purpose of this article is to introduce proximity structures using general binary relation in approximation space (X, R).

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Let X be a non-empty finite set called the universe. A relation R from a universe X to a universe X (relation on X) is a subset of $X\times X$, i.e., $R\subseteq X\times X$. The formula $(x, y)\in R$ is abbreviated as xRy which means that x related to y via relation R. The pair (X, R) is called a generalized approximation space.

Definition 3.1 [4] If R is a relation on X, then the right neighborhood (aftersets) of $x\in X$ is xR, where $xR=\{y\in X : xRy\}$ and the left neighborhood (forsets) of $x\in X$ is Rx, where $Rx=\{y\in X : yRx\}$.

Definition 3.2 [1] For any binary relation R on X, then $\langle x\rangle R=\cap\{yR : x\in yR\}$ and $R\langle x\rangle=\cap\{Ry : x\in Ry\}$.

Lemma 3.3 [1] For any binary relation R on X if $x\in\langle y\rangle R$, then $\langle x\rangle R\subseteq\langle y\rangle R$.

Lemma 3.4. For a symmetric and transitive relation R on X, then $yR\subseteq\langle y\rangle R$.

Proof Let $x\notin\langle y\rangle R$, then $\exists z\in X$ s.t. $x\notin zR$ and $y\in zR$. Since R is symmetric and transitive relation on X, then $x\notin yR$.

Corollary 3.5 For an equivalence relation R on X, then $\forall x\in X$; $xR=\langle x\rangle R$.

Definition 3.6 [1] Let R be a reflexive relation on X, then the class $\{\langle x\rangle R : x\in X\}$ (resp., $\{R\langle x\rangle : x\in X\}$) is a base for the topology on X.

Definition 3.7 Let (X, R) be an approximation space. Then, $\forall A\subseteq X$, the operations $R_1A=\{x\in X : \langle x\rangle R\cap A\neq\varnothing\}$ and $R_2A=\{x\in X : R\langle x\rangle\cap A\neq\varnothing\}$.

The proof of the following lemmas are obvious and omitted.

Lemma 3.8 For any binary relation R on X we have, $x\in R_1\{y\}\leftrightarrow y\in\langle x\rangle R$ (resp., $x\in R_2\{y\}\leftrightarrow y\in R\langle x\rangle$).

Lemma 3.9 Let R be any binary relation on X. Then, $\forall A\subseteq X$, $R_1A=\cup_{x\in A}R_1\{x\}$(resp., $R_2A=\cup_{x\in A}R_2\{x\}$).

The following theorem indicates some properties of R_1A and R_2A for any $A\subseteq X$.

Theorem 3.10 In an approximation space (X, R), let A and B be two subsets of X. Then, the following properties hold:
1) $R_1\varnothing=R_2\varnothing=\varnothing$.
2) If R is reflexive, then $R_1X=R_2X=X$.
3) If $A\subseteq B$, then $R_1A\subseteq R_1B$ and $R_2A\subseteq R_2B$.
4) $R_1(A\cup B)=R_1A\cup R_1B$ and $R_2(A\cup B)=R_2A\cup R_2B$.
5) $R_1(A\cap B)\subseteq R_1A\cap R_1B$ and $R_2(A\cap B)\subseteq R_2A\cap R_2B$.
6) If R is reflexive, then $A\subseteq R_1A$ and $A\subseteq R_2A$.
7) If R is reflexive, then $R_1R_1A=R_1A$ and $R_2R_2A=R_2A$.
8) If R is symmetric, then $R_1A=R_2A$.

Proof We shall prove only (7) for R_1 and other statements are obvious from Definition of R_1 and R_2. Since R_1 is reflexive and by using (2), then $R_1A\subseteq R_1R_1A$. The other side,

$$\begin{aligned} x\in R_1R_1A &\rightarrow \langle x\rangle R\cap R_1A\neq\varnothing \\ &\rightarrow \exists y\in\langle x\rangle R \text{ such that } y\in R_1A. \\ &\rightarrow \langle y\rangle R\subseteq\langle x\rangle R \text{ and } \langle y\rangle R\cap A\neq\varnothing. \\ &\rightarrow \langle x\rangle R\cap A\neq\varnothing \rightarrow x\in R_1A. \end{aligned}$$

Definition 3.11 Let X be any set and $R\subseteq X\times X$ be any binary relation on X. The relation R gives rise to a closure operation $Cl_{\delta1}A$ and $Cl_{\delta2}A$ on X as follows: $Cl_{\delta1}A=A\cup R_1A$ (resp.,$Cl_{\delta2}A=A\cup R_2A$).

Definition 3.12 Let X be any set and $R\subseteq X\times X$ be a reflexive relation on X. The relation R gives rise to a closure operation $Cl_{\delta1}A$ and $Cl_{\delta2}A$ on X as follows: $Cl_{\delta1}A= R_1A$ (resp.,$Cl_{\delta2}A= R_2A$).

Definition 3.13 Let $X\neq\varnothing$; and $A\subseteq X$, the relations δ_1 and δ_2 on P(X) generated from R_1 and R_2 respectively are defined as follows, $x\delta_1A\leftrightarrow x\in Cl_{\delta1}A$ (resp., $x\delta_2A\leftrightarrow x\in Cl_{\delta2}A$).

Lemma 3.14 Let X be any set and $R\subseteq X\times X$ be any binary relation on X. If $\langle x\rangle R=\varnothing$ then $\{x\}$ is $\tau_{\delta1}$-closed (resp., If $R\langle x\rangle=\varnothing$, then $\{x\}$ is $\tau_{\delta2}$-closed).

Proof Obvious.

In the following theorem, some properties of δ_1 and δ_2 which generated from R_1 and R_2 are obtained.

Theorem 3.15 Let $X\neq\varnothing$; R be a binary relation on X and δ_1 and δ_2 generated from R_1 and R_2 respectively, then :
i) δ_1 and δ_2 satisfy P1 and P2 axioms of Definition 2.1. for any relation R.
ii) δ_1 and δ_2 satisfy P1, P2 and P3 axioms of Definition 2.1. for a reflexive relation R.
iii) δ_1 and δ_2 are K-proximities on X for equivalence relation R.

Proof i) From (1) and (4) in Theorem 3.10., then δ_1 and δ_2 satisfy P1 and P2 axioms of Definition 2.1.
ii) From (1), (4) and (6) in Theorem 3.10., then δ_1 and δ_2 satisfy P1, P2 and P3 axioms of Definition 2.1.
iii) Obvious in view of Theorem 3.10. and Lemmas 3.3., 3.8.
We shall prove only δ_1 is K-proximity on X. Since R is reflexive, then the axioms P1, P2 and P3 of Definition 2.1. are hold. Let $x\delta_1^{-}A$, then $x\notin R_1A$. By using Corollary 3.5., Def. 3.7. and (6), (7) of Theorem 3.10., then $x\notin A$ and $x\notin R_1R_1A$. Hence $\langle x\rangle R\cap R_1A=\varnothing$. Suppose $E=R_1A$, then $\forall y\in xR$; $y\notin E$ i.e. $y\in(X-E)$. Since R is symmetric, then $x\in yR$ and $yR\cap A=\varnothing$. This implies that $x\notin R_1E$ and $y\notin R_1A$ $\forall y\in(X-E)$. Consequently, $x\delta_1^{-}E$ and $y\delta_1^{-}A$ $\forall y\in(X-E)$.

Theorem 3.16 Let R be a binary relation on X. If $\langle x\rangle R=\varnothing$ $\forall x\in X$, then δ_1 and δ_2 are separated proximity.

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Proof Obvious in view of Lemma 3.14.

In the following, we use the deviation between R_1A, R_2A to construct classifications for P(X).

Definition 3.17 Let (X, R) be an approximation space. If $A \subseteq X$, then:
i) A is R-certain set if $R_1A=A=R_2A$.
ii) A is R-right certain and left uncertain set if $A=R_1A$, $A\neq R_2A$.
iii) A is R-left certain and right uncertain set if $A\neq R_1A$, $A=R_2A$.
iv) A is R-uncertain set if $A\neq R_1A$, $A\neq R_2A$.

Definition 3.18 [1] If τ is the topology on a finite set X and the class $\tau^c= \{(X - G): G\in\tau\}$ is also the topology on X, then τ^c is the dual of τ .

Definition 3.19 [16] The proximities δ_1 and δ_2 on the power set of X are dual iff $A\delta_2B$ is equivalent to $A\delta_1^{-}$ (X-B), $\forall A, B\subseteq X$.

Theorem 3.20 If R be a symmetric relation on a non-empty set X, then $\delta_1=\delta_2$, $\tau_{\delta1}=\tau_{\delta2}$ and $\tau_1=\tau_2$.

The following Examples illustrate the form and relations of δ_1 and δ_2 and summarize these results in tables 1- 8. Also, we shall illustrate the form and relations of $\tau_{\delta1}$ and $\tau_{\delta2}$ which generated from δ_1 and δ_2.

Example 1 Let X={a, b, c, d} and R={(a, a), (a, b), (b, d), (c, d), (d, a)} be any relation on X where aR={a, b}, bR=cR={d},dR={a} and Ra={a, d}, Rb={a}, Rc=∅, Rd={b, c}. Also, <a>R={a},<b>R={a, b}, <c>R=∅, <d>R={d} and R<a>={a}, R<b>=R<c>={b, c}, R<d>={a, d}.

Table 1

	$\Re_1A$	$\Re_2A$
A_1={a}	{a, b}	{a, d}
A_2={b}	{b}	{b, c}
A_3={c}	∅	{b, c}
A_4={d}	{d}	{d}
A_5={a, b}	{a, b}	X
A_6={a, c}	{a, b}	X
A_7={a, d}	{a, b, d}	{a, d}
A_8={b, c}	{b}	{b, c}
A_9={b, d}	{b, d}	{b, c, d}
A_{10}={c, d}	{d}	{b, c, d}
A_{11}={a, b, c}	{a, b}	X
A_{12}={a, b, d}	{a, b, d}	X
A_{13}={a, c, d}	{a, b, d}	X
A_{14}={b, c, d}	{b, d}	{b, c, d}
A_{15}=∅	∅	∅
A_{16}=X	{a, b, d}	X

The form of proximity relations δ_1 and δ_2 are obtained from the table 1 and Definition 3.13. It is clear that the topology $\tau_{\delta1}$={∅, X, {a},{c}, {d}, {a; b}, {a; c}, {a; d}, {c; d}, {a; c; d}, {a; b; c}, {a; b; d}} which is generated from δ_1 and the topology $\tau_{\delta2}$={∅, X, {a}, {b; c}, {a; d}, {a; b; c}} which is generated from δ_2. The topologies τ_1 and τ_2 generated from the relation R, τ_1 ={∅, X, {a; b}, {a; d}, {d}, {a}, {a; b; d}}$\subseteq\tau_{\delta1}$ and $\tau_2 =\tau_{\delta2}$.

Corollary 3.21 It is clear that from Example 1 A_4, A_{15} are R-certain set. A_7, A_8, A_{14}, A_{16} are R-left certain and right uncertain set. A_2, A_5, A_9, A_{12} are R-right certain and left uncertain set but others are R-uncertain sets.

Corollary 3.22 It is clear that from Example 1 δ_1 is not dual δ_2 and $\tau_{\delta1}$ is not the dual $\tau_{\delta2}$. Generally for any binary relation R, then τ_1 is not the dual of τ_2.

Now we introduce some Examples about special cases of the relation R to study the duality between proximities δ_1 and δ_2 the duality between topologies τ_1 and τ_2.

Example 2 Let X={a, b, c, d}, and R be a reflexive on X, R={(a, a), (b,b), (c, c), (d, d), (a, b), (b, d), (c, a)} where aR={a, b}, bR={b, d}, cR={a, c}, dR={d} and Ra={a, c}, Rb={a, b}, Rc={c}, Rd={b, d}. <a>R={a},<b>R={b}, <c>R={a, c}, <d>R={d} and R<a>={a}, R<b>={b}, R<c>={c}, R<d>={b, d}.

Table 2

	$\Re_1A$	$\Re_2A$
A_1={a}	{a, c}	{a}
A_2={b}	{b}	{b, d}
A_3={c}	{c}	{c}
A_4={d}	{d}	{d}
A_5={a, b}	{a, b, c}	{a, b, d}
A_6={a, c}	{a, c}	{a, c}
A_7={a, d}	{a, c, d}	{a, d}
A_8={b, c}	{b, c}	{b, c, d}
A_9={b, d}	{b, d}	{b, d}
A_{10}={c, d}	{c, d}	{c, d}
A_{11}={a, b, c}	{a, b, c}	X
A_{12}={a, b, d}	X	{a, b, d}
A_{13}={a, c, d}	{a, c, d}	{a, c, d}
A_{14}={b, c, d}	{b, c, d}	{b, c, d}
A_{15}=∅	∅	∅
A_{16}=X	X	X

The form of proximity relations δ_1 and δ_2 are obtained from the table 2 and Definition 3.13. It is clear that the topology $\tau_{\delta1}$ ={∅, X, {d}, {a}, {b}, {b; d}, {a; b}, {a; d}, {a; c}, {a; b; c}, {a; b; d}, {a; c; d}} generated by δ_1 and the topology $\tau_{\delta2}$ ={∅, X, {c}, {a}, {b}, {b; d}, {a; b}, {b; c}, {a; c}, {a; b; c}, {a; b; d}, {b; c; d}} which is generated from δ_2. The topologies τ_1 and τ_2 generated from the relation R, $\tau_1=\tau_{\delta1}$and $\tau_2=\tau_{\delta2}$.

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Corollary 3.23 It is clear that from Example 2 A_3, A_4, A_6, A_9, A_{10}, A_{13}, A_{14}, A_{15} and A_{16} are R-certain sets. A_1, A_7 and A_{12} are R-left certain and right uncertain sets. A_2, A_8 and A_{11} are R-right certain and left uncertain sets but others are R-uncertain sets.

Example 3 Let X={a, b, c, d}, and R be a symmetric on X, R={(a, a), (b, b), (a, b), (b, a), (c, d), (d, c), (a, c), (c, a)} where aR=Ra={a, b, c}, bR=Rb= {b, a}, cR=Rc={a, d}, dR=Rd={c}. <a>R=R<a>={a},<b>R=R<b>= {a, b}, <c>R=R<c>={c}, <d>R=R<d>={a, d}.

Table 3

	$\mathfrak{R}_1A$	$\mathfrak{R}_2A$
A_1={a}	{a, b, d}	{a, b, d}
A_2={b}	{b}	{b}
A_3={c}	{c}	{c}
A_4={d}	{d}	{d}
A_5={a, b}	{a, b, d}	{a, b, d}
A_6={a, c}	X	X
A_7={a, d}	{a, b, d}	{a, b, d}
A_8={b, c}	{b, c}	{b, c}
A_9={b, d}	{b, d}	{b, d}
A_{10}={c, d}	{c, d}	{c, d}
A_{11}={a, b, c}	X	X
A_{12}={a, b, d}	{a, b, d}	{a, b, d}
A_{13}={a, c, d}	X	X
A_{14}={b, c, d}	{b, c, d}	{b, c, d}
A_{15}=∅	∅	∅
A_{16}=X	X	X

The form of proximity relations δ_1 and δ_2 are obtained from the table 3 and Definition 3.13. $\tau_{\delta 1}=\tau_{\delta 2}$={∅, X, {a; b}, {a; d}, {a; c}, {c}, {a}, {a; b; d}, {a; b; c}, {a; c; d}}. The topologies τ_1 and τ_2 generated from the relation R, $\tau_{\delta 1}=\tau_{\delta 2}=\tau_1=\tau_2$.

Corollary 3.24 It is clear that from Example 3 the topology $\tau_{\delta 1}=\tau_{\delta 2}$ generated by δ_1 and δ_2 respectively, $\tau_{\delta 1}$ is the dual $\tau_{\delta 2}$.

Corollary 3.25 It is clear that from Example 3 A_2, A_3, A_4, A_8, A_9, A_{10}, A_{12}, A_{14}, A_{15} and A_{16} are R-certain sets but others are R-uncertain sets.

Example 4 Let X= {a, b, c, d}, and R be a transitive on X, R= {(a, a), (a,b), (b, a), (b, b), (c, d), (d, a), (c, a), (d, b), (c, b)} where aR=bR=dR={a, b}, cR={a, b, d} and Ra=Rb=X, Rc=∅, Rd={c}. <a>R= <b>R={a, b}, <c>R=∅, <d>R= {a, b, d} and R<a>=R<b>=R<d>=X, R<c>={c}.

The form of proximity relations δ_1 and δ_2 are obtained from the table 4 and Definition 3.13. $\tau_{\delta 1}$={∅, X, {a; b}, {c}, {a; b; d}, {a; b; c}} and $\tau_{\delta 2}$={∅, X, {c}}. The topologies τ_1 and τ_2 generated from the relation R are τ_1={∅;X; {a; b}; {a; b; d}}$\subseteq\tau_{\delta 1}$ and $\tau_2=\tau_{\delta 2}$.

Table 4

	$\mathfrak{R}_1A$	$\mathfrak{R}_2A$
A_1={a}	{a, b, d}	{a, b, d}
A_2={b}	{a, b, d}	{a, b, d}
A_3={c}	∅	X
A_4={d}	{d}	{a, b, d}
A_5={a, b}	{a, b, d}	{a, b, d}
A_6={a, c}	{a, b, d}	X
A_7={a, d}	{a, b, d}	{a, b, d}
A_8={b, c}	{a, b, d}	X
A_9={b, d}	{a, b, d}	{a, b, d}
A_{10}={c, d}	{d}	X
A_{11}={a, b, c}	{a, b, d}	X
A_{12}={a, b, d}	{a, b, d}	{a, b, d}
A_{13}={a, c, d}	{a, b, d}	X
A_{14}={b, c, d}	{a, b, d}	{b, c, d}
A_{15}=∅	∅	∅
A_{16}=X	{a, b, d}	X

Corollary 3.26 It is clear that from Example 4 A_{12} and A_{15} are R-certain set. A_{16} is R-left certain and right uncertain set. A_4 is R-right certain and left uncertain set but others are R-uncertain sets.

Corollary 3.27 It is clear that from Example 4 $\tau_{\delta 2}\subseteq\tau_{\delta 1}$ i.e. $\tau_{\delta 1}$ is not the dual $\tau_{\delta 2}$.

Example 5 Let X={a, b, c, d}, and R be a reflexive and transitive on X, R={(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (a, c), (c, d), (a, d),(b, c), (b, d)} where aR=bR=X, cR={c, d}, dR={d} and Ra=Rb={a, b},Rc={a, b, c}, Rd=X. <a>R=<b>R=X, <c>R={c, d}, <d>R={d} andR<a>=R<b>={a, b}, R<c>={a, b, c}, R<d>=X .

The form of proximity relations δ_1 and δ_2 are obtained from the table 5 and Definition 3.13. The topology $\tau_{\delta 1}$= {∅, X, {d}, {c; d}} generated by δ_1 and the topology $\tau_{\delta 2}$={∅, X, {a; b}, {a; b; c}} generated by δ_2. The topologies τ_1 and τ_2 generated from the relation R, are $\tau_1=\tau_{\delta 1}$ and $\tau_2=\tau_{\delta 2}$. Also, we have $\tau_1=\tau^c_2$.

Corollary 3.28 It is clear that from Example 5. A_{15} and A_{16} are R-certain sets. A_4 and A_{10} are R-left certain and right uncertain sets. A_5 and A_{11} are R-right certain and left uncertain sets but others are R-uncertain sets.

Corollary 3.29 It is clear that from Example 5 the union of $\tau_{\delta 1}$ and $\tau_{\delta 2}$ is a quasi discrete topological spaces (clopen topology).

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Table 5

	$\Re_1 A$	$\Re_2 A$
$A_1=\{a\}$	{a, b}	X
$A_2=\{b\}$	{a, b}	X
$A_3=\{c\}$	{a, b, c}	{c, d}
$A_4=\{d\}$	X	{d}
$A_5=\{a, b\}$	{a, b}	X
$A_6=\{a, c\}$	{a, b, c}	X
$A_7=\{a, d\}$	X	X
$A_8=\{b, c\}$	{a, b, c}	X
$A_9=\{b, d\}$	X	X
$A_{10}=\{c, d\}$	X	{c, d}
$A_{11}=\{a, b, c\}$	{a, b, c}	X
$A_{12}=\{a, b, d\}$	X	X
$A_{13}=\{a, c, d\}$	X	X
$A_{14}=\{b, c, d\}$	X	X
$A_{15}=\varnothing$	$\varnothing$	$\varnothing$
$A_{16}=X$	X	X

Example 6 Let X={a, b, c, d}, and R be a symmetric and transitive on X, R={(a, a), (b, b), (c, c), (a, b), (b, a), (b, c), (c, b), (a, c), (c, a)} whereaR=bR=cR=Ra=Rb=Rc={a, b, c}, dR=Rd=∅. <a>R=R<a>=<b>R=R<b>=<c>R=R<c>= {a, b, c}, <d>R=R<d>=∅.

Table 6

	$\Re_1 A$	$\Re_2 A$
$A_1=\{a\}$	{a, b, c}	{a, b, c}
$A_2=\{b\}$	{a, b, c}	{a, b, c}
$A_3=\{c\}$	{a, b, c}	{a, b, c}
$A_4=\{d\}$	$\varnothing$	$\varnothing$
$A_5=\{a, b\}$	{a, b, c}	{a, b, c}
$A_6=\{a, c\}$	{a, b, c}	{a, b, c}
$A_7=\{a, d\}$	{a, b, c}	{a, b, c}
$A_8=\{b, c\}$	{a, b, c}	{a, b, c}
$A_9=\{b, d\}$	{a, b, c}	{a, b, c}
$A_{10}=\{c, d\}$	{a, b, c}	{a, b, c}
$A_{11}=\{a, b, c\}$	{a, b, c}	{a, b, c}
$A_{12}=\{a, b, d\}$	{a, b, c}	{a, b, c}
$A_{13}=\{a, c, d\}$	{a, b, c}	{a, b, c}
$A_{14}=\{b, c, d\}$	{a, b, c}	{a, b, c}
$A_{15}=\varnothing$	$\varnothing$	$\varnothing$
$A_{16}=X$	{a, b, c}	{a, b, c}

The form of proximity relations δ_1 and δ_2 are obtained from the table 6 and Definition 3.13. The topologies $\tau_{\delta 1} = \tau_{\delta 2}= \{\varnothing, X, \{a; b; c\}, \{d\}\}$ generated by δ_1 and δ_2. The topologies τ_1 and τ_2 generated from the relation R, are $\tau_1=\tau_2= \{\varnothing, X, \{a; b; c\}\} \subseteq \tau_{\delta 1} =\tau_{\delta 2}$.

Corollary 3.30 It is clear that from Example 6. A_{11} and A_{15} are R-certain sets, but others are R-uncertain sets.

Corollary 3.31 It is clear that from Example 6 the union of $\tau_{\delta 1} =\tau_{\delta 2}$ is a quasi discrete topological space (clopen topology).

Example 7 Let X={a, b, c, d}, and R be a reflexive and symmetric on X, R={(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (a, c), (c, a), (b, d), (d, b)} where aR=Ra={a, b, c}, bR=Rb={a, b, d}, cR=Rc= {a, c}, dR=Rd={b, d}. <a>R=R<a>={a},<b>R=R<b>={b}, <c>R=R<c>={a, c}, <d>R=R<d>={b, d}.

Table 7

	$\Re_1 A$	$\Re_2 A$
$A_1=\{a\}$	{a, c}	{a, c}
$A_2=\{b\}$	{b, d}	{b, d}
$A_3=\{c\}$	{c}	{c}
$A_4=\{d\}$	{d}	{d}
$A_5=\{a, b\}$	X	X
$A_6=\{a, c\}$	{a, c}	{a, c}
$A_7=\{a, d\}$	{a, c, d}	{a, c, d}
$A_8=\{b, c\}$	{b, c, d}	{b, c, d}
$A_9=\{b, d\}$	{b, d}	{b, d}
$A_{10}=\{c, d\}$	{c, d}	{c, d}
$A_{11}=\{a, b, c\}$	X	X
$A_{12}=\{a, b, d\}$	X	X
$A_{13}=\{a, c, d\}$	{a, c, d}	{a, c, d}
$A_{14}=\{b, c, d\}$	{b, c, d}	{b, c, d}
$A_{15}=\varnothing$	$\varnothing$	$\varnothing$
$A_{16}=X$	X	X

The form of proximity relations δ_1 and δ_2 are obtained from the table 7 and Definition 3.13. $\tau_{\delta 1} = \tau_{\delta 2} = \{\varnothing, X, \{a; b; d\}, \{a; b;c\}, \{b; d\}, \{a; b\}, \{a; c\}, \{a\},\{b\}\}$. The topologies τ_1 and τ_2 generated from the relation R are $\tau_2 =\tau_1 =\tau_{\delta 2} =\tau_{\delta 1}$.

Corollary 3.32 It is clear that from Example 7 A_3, A_4, A_6, A_9, A_{10}, A_{13}, A_{14}, A_{15} and A_{16} are R-certain sets, but others are R-uncertain sets.

Example 8 Let X={a, b, c, d}, and R be an equivalence relation on X, R={(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (a, c), (c, a), (a, d), (d, a), (b,d), (b, c), (d, b), (c, b), (c, d), (d, c)} where aR=bR=cR=dR=Ra=Rb=Rc=Rd=X. <a>R=R<a>=<b>R=R<b>=<c>R=R<c>=<d>R=R<d>=X.

The form of proximity relations δ_1 and δ_2 are obtained from the table 8 and Definition 3.13. The topology $\tau_{\delta 1} = \tau_{\delta 2} =\{\varnothing, X\}$ generated by δ_1 and δ_2. The topologies τ_1 and τ_2 generated from the relation R defined in Example 8. are $\tau_1=\tau_2 =\tau_{\delta 1} =\tau_{\delta 2}$.

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Corollary 3.33 It is clear that from Example 8 A_{15} and A_{16} are R-certain sets but others are R-uncertain sets.

The topologies $\tau_{\delta 1}$ and $\tau_{\delta 2}$ generated by δ_1 respectively δ_2 are indiscrete.

Corollary 3.34 It is clear that from Example 8 the union of $\tau_{\delta 1}$ and $\tau_{\delta 2}$ is a quasi discrete topological space (clopen topology).

Corollary 3.35 It is clear that $\tau_{\delta 1} = \tau_1$ and $\tau_{\delta 2} = \tau_2$. Also, we have $\tau_1 = \tau_2^c$.

Corollary 3.36 Obviously, if R is an equivalence relation, then <x>R definition is equivalent to original Pawlak s Definition [13]

Table 8

	$\Re_1 A$	$\Re_2 A$
$A_1=\{a\}$	X	X
$A_2=\{b\}$	X	X
$A_3=\{c\}$	X	X
$A_4=\{d\}$	X	X
$A_5=\{a, b\}$	X	X
$A_6=\{a, c\}$	X	X
$A_7=\{a, d\}$	X	X
$A_8=\{b, c\}$	X	X
$A_9=\{b, d\}$	X	X
$A_{10}=\{c, d\}$	X	X
$A_{11}=\{a, b, c\}$	X	X
$A_{12}=\{a, b, d\}$	X	X
$A_{13}=\{a, c, d\}$	X	X
$A_{14}=\{b, c, d\}$	X	X
$A_{15}=\varnothing$	$\varnothing$	$\varnothing$
$A_{16}=X$	X	X

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